**ZiriTadi koncefciebi rekursiebze**

mravali funqcia savsebiT bunebrivad gani­sazRvreba sxva funqciaTa gamoyenebiT. magaliTad, SesaZlebelia ganvsazRvroT funqcia, romelic gvibrunebs arauaryofiTi mTeli ricxvis faqtorials sabibli­oTeko funqciebis gamoyenebiT erTsa da mocemul mniSvnelobas Soris mo­Tav­sebul ricxvTa namravlis gamosaTvlelad:

*factorial* :: *Int → Int*

*factorial n* = *product* [1*..n* ]

haskelSi daSvebulia agreTve funqciaTa gansazRvra sakuTari Tavis gamoyenebiT. aseT SemTxvevaSi funqciebs *rekursiuls* uwodeben. magaliTad, am gziT SeiZleba ganisazRvros *factorial* funqciac:

*factorial* 0 = 1

*factorial* (*n* + 1) = (*n* + 1)∗ *factorial n*

pirveli gantoleba gveubneba, rom nulis faqtoriali erTia da am gantolebas *sabazo* (*sayrdeni*) *SemTxveva* (*gamosaxuleba*) ewodeba. meore gantoleba amtkicebs, rom nebismieri mkacrad dadebiTi mTeli ricxvis faqtoriali warmoadgens am ricxvisa da misi winamavali ricxvis faqtorialis namravls. mas *rekursiuli SemTxveva* (*gamosaxuleba*) ewodeba.

magaliTad, Semdegi fragmenti gviCvenebs, Tu rogor xdeba samis faqtorialis gamoTvla am gansazRvrebis gamoyenebiT:

*factorial* 3

= { *factorial* funqciis gamoyeneba}

3 ∗ *factorial* 2

= { *factorial* funqciis gamoyeneba}

3 ∗ (2 ∗ *factorial* 1)

= { *factorial* funqciis gamoyeneba}

3 ∗ (2 ∗(1 ∗ *factorial* 0))

= { *factorial* funqciis gamoyeneba}

3 ∗ (2 ∗(1 ∗1))

= {∗ operaciis gamoyeneba}

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yuradReba miaqcieT im garemoebas, rom, Tumca *factorial* funqcia sakuTari Ta­vis saSualebiT ganisazRvreba, igi usasrulo cikls ar warmoadgens. sa­xel­dobr, *factorial* funqciis yoveli gamoyeneba amcirebs mTelricxva ar­guments erTiT, vidre igi saboloo jamSi nulis toli ar gaxdeba. aq re­kursia wydeba da gamravlebis operacia sruldeba. nulis faqtoriali, rom­liTac erTiani gvibrundeba, savsebiT adekvaturia am viTarebaSi, rad­gan gamravlebisas erTiani igiveobas unarCunebs gamosaxulebas. sxvanairad rom vTqvaT, 1 ∗*x*= *x* da *x*∗1 = *x* nebismieri mTeli *x* ricxvisaTvis.

*factorial* funqciis SemTxvevaSi sawyisi gansazRvreba sabiblioTeko funqciebis saSualebiT ufro martivia, vidre rekursiis gamoyenebiT. magram, rogorc amas wignis darCenil nawilSi davinaxavT, mravali funqcia martivad da bunebrivad swored rekursiis gamoyenebiT ganisazRvreba. magaliTad, mravali sabiblioTeko funqcia haskelSi ganisazRvreba rekursiiT. garda amisa, rogorc me-13 TavSi vnaxavT, funqciaTa gansazRvra rekursiiT maTi Tvisebebis damtkicebis saSualebas iZleva maTematikuri induqciis mZlavri meTodis gamoyenebiT.

mTel ricxvebze rekursiis kidev erT magaliTad ganvixiloT zemoT gamoyenebuli gamravlebis ∗ operatori. efeqturobis mosazrebebidan gamomdinare, haskelSi es operatori gaTvaliswinebulia rogorc primitivi \_ daprogramebis enaSi programaTa Sesrulebis siCqaris gazrdisaTvis CaSenebuli operatori. magram arauaryofiTi mTeli ricxvebisaTvis igi SeiZleba iyos gansazRvruli aseve rekursiiTac Tavisi ori argumentidan nebismier erTze, vTqvaT meoreze:

(∗) :: *Int → Int → Int*

*m* ∗0 = 0

*m* ∗(*n* + 1) = *m* + (*m* ∗ *n*)

magaliTad:

4 ∗3

= {∗opratoris gamoyeneba }

4 + (4 ∗2)

= {∗opratoris gamoyeneba }

4 + (4 + (4 ∗1))

= {∗opratoris gamoyeneba }

4 + (4 + (4 + (4 ∗ 0)))

= { ∗opratoris gamoyeneba }

4 + (4 + (4 + 0))

= {+ opratoris gamoyeneba }

12

amrigad, aq rekursiuli gansazRvreba ∗ operatorisaTvis formalurad gamoxatavs im ideas, rom gamravleba daiyvaneba iteraciul (mravaljerad) Sekrebamde.

**rekursia siebze**

rekursia ar Semoifargleba funqciebiT mTel ricxvebze, igi SeiZleba aseve gamoyenebuli iqnes funqciaTa gansazRvrisaTvis siebze. magaliTad, *product* sabiblioTeko funqcia, romelic wina punqtSi vixmareT, Semdegi saxiT SeiZleba ganisazRvros:

*product* :: *Num a* ⇒[a] *→ a*

*product* [] = 1

*product* (*n* : *ns*) = *n* ∗ *product ns*

pirveli gantoleba amtkicebs, rom carieli siis namravli erTiania, rac savsebiT adekvaturia, radgan erTiani gamravlebis operaciaSi unarCunebs gamosaxulebas igiveobas. meore gantoleba ki gveubneba, rom nebismieri aracarieli siis namravli miiReba pirveli ricxvisa da ricxvTa darCenili siis namravlis erTmaneTze gamravlebis operaciiT. magaliTad:

*product* [2*,* 3*,* 4]

= { *product* funqciis gamoyeneba }

2 *∗ product* [3*,* 4]

= { *product* funqciis gamoyeneba }

2 ∗(3 ∗ *product* [4])

= { *product* funqciis gamoyeneba }

2 ∗(3 ∗ (4 ∗ *product* []))

= { *product* funqciis gamoyeneba }

2 ∗(3 ∗(4 ∗1))

= {∗operatoris gamoyeneba }

gavixsenoT, rom realurad sias haskelSi aqvs erTi elementis logikuri struqtura, romelic amave dros cons operators iyenebs. maSasadame, [2*,* 3*,* 4] Canaweri mxolod abreviaturaa 2 : (3 : (4 : [])) gamosaxulebisaTvis da meti araferi. ganvixiloT siebze rekursiis kidev erTi martivi magaliTi, risTvisac mivmarToT *length* sabiblioTeko funqcias, romelic SeiZleba ganisazRvros rekursiis amave Sablonis gamoyenebiT *product* funqciis msgavsad:

*length* :: [*a*] *→ Int*

*length* [] = 0

*length* ( \_ : *xs*) = 1 + *length xs*

maSasadame, carieli siis sigrZe nulia, xolo nebismieri aracarieli siis sigrZe misi kudis sigrZis momdevno mniSvnelobaa. yuradReba miaqcieT ( \_ ) Casmis Sablonis gamoyenebas rekursiul gamosaxulebaSi. es Sabloni asaxavs im faqts, rom siis sigrZe damokidebuli ar aris misi elementebis mniSvnelobaze.

axla ganvixiloT sabiblioTeko funqcia, romelic siis Sebrunebas, Seqcevas (reverss) axorcielebs. rekursiis saSualebiT es funqcia SeiZleba Semdegnairad ganvsazRvroT:

*reverse* :: [*a*] *→* [*a*]

*reverse* [] = []

*reverse* (*x* : *xs*) = *reverse xs* ++ [*x*]

maSasadame, carieli siis reversi kvlav carieli siaa, xolo nebismieri aracarieli siis reversi xorcieldeba mibmiT misi kudis reversTan erTelementiani siis, romelic sawyisi siis Tavs warmoadgens. magaliTad:

*reverse* [1*,* 2*,* 3]

= { *reverse* funqciis gamoyeneba }

*reverse* [2*,* 3] ++ [1]

= { *reverse* funqciis gamoyeneba }

(*reverse* [3] ++ [2])++ [1]

= { *reverse* funqciis gamoyeneba }

((*reverse* []++ [3])++ [2])++ [1]

= { *reverse* funqciis gamoyeneba }

(([] ++ [3])++ [2]) ++ [1]

= {++ operatoris gamoyeneba }

[3*,* 2*,* 1]

Tavis mxriv, damatebis ++ operatori, romelic *reverse* funqciis zemoT mocemul gansazRvrebaSi gamoiyeneba, Tavad SeiZleba iqnes aRwerili rekursiiT Tavis pirvel argumentze:

(++) :: [*a*] *→* [*a*] *→* [*a*]

[]++ *ys* = *ys*

(*x* : *xs*) ++ *ys* = *x* : (*xs* ++ *ys*)

magaliTad:

[1*,* 2*,* 3] ++ [4*,* 5]

= {++ operatoris gamoyeneba }

1 : ([2*,* 3] ++ [4*,* 5])

= { ++ operatoris gamoyeneba }

1 : (2 : ([3]++ [4*,* 5]))

= {++ operatoris gamoyeneba }

1 : (2 : (3 : ([] ++ [4*,* 5])))

= {++ operatoris gamoyeneba }

1 : (2 : (3 : [4*,* 5]))

= {Cawera siis saxiT }

[1*,* 2*,* 3*,* 4*,* 5]

amrigad, rekursiuli gansazRvreba ++ operatorisaTvis formalurad asaxavs im ideas, rom ori siis gadabma SeiZleba pirveli siidan elementebis piris (aslis) gadaRebiT, vidre es sia ar amoiwureba, ris Semdeg meore sia daemateba am asls boloSi.

ganvixiloT kidev ori magaliTi, romelic exeba rekursias daxarisxebul siebze. upirveles yovlisa, ganvixiloT funqcia, romelic daxarisxebul siaSi axorcielebs nebismieri mowesrigebuli tipis axali elementis Casmas kidev erTi axali daxarisxebuli siis misaRebad. es funqcia Semdegi saxiT SeiZleba ganisazRvros:

*insert* :: *Ord a* ⇒ *a →* [*a*] *→* [*a*]

*insert x* [] = [*x*]

*insert x* (*y* : *ys*) *| x* ≤ *y* = *x* : *y* : *ys*

*| otherwise* = *y* : *insert x ys*

maSasadame, axali elementis Casma cariel siaSi iZleva erTelementian sias, maSin roca aracarieli siisaTvis Sedegi damokidebulia axali *x* elementisa da siis *y* Tavis urTierTganlagebaze. saxeldobr, Tu *x* ≤ *y*, maSin axali *x* elementi mxolod Tavsdeba siis dasawyisSi da meti araferi, winaaRmdeg SemTxvevaSi *y* Tavi saboloo siis pirveli elementi xdeba da Semdeg iwyeben axali elementis Casmas mocemuli siis kudSi. magaliTad:

*insert* 3 [1*,* 2*,* 4*,* 5]

= { *insert* funqciis gamoyeneba }

1 : *insert* 3 [2*,* 4*,* 5]

= { *insert* funqciis gamoyeneba }

1 : 2 : *insert* 3 [4*,* 5]

= { *insert* funqciis gamoyeneba }

1 : 2 : 3 : [4*,* 5]

= {Cawera siis saxiT }

[1*,* 2*,* 3*,* 4*,* 5]

*insert* funqciis gamoyenebiT SesaZlebelia axali funqciis gansazRvra, romelic axor­cielebs *daxarisxebas CasmiT*[[1]](#footnote-1) (ingl. *insertion sort*). saxeldobr, am gansa­zRvre­baSi unda iqnes gaTvaliswinebuli, rom carieli sia ukve daxarisxebulia, xolo nebismieri aracarieli siis daxarisxeba xdeba misi Tavis CasmiT kudis daxarisxebis Sedegad miRebul siaSi:

*isort* :: *Ord a* ⇒ [a] *→* [a]

*isort* [] = []

*isort* (*x* : *xs*) = *insert x* (*isort xs*)

magaliTad:

*isort* [3*,* 2*,* 1*,* 4]

= { *isort* funqciis gamoyeneba }

*insert* 3 (*insert* 2 (*insert* 1 (*insert* 4 [])))

= { *insert* funqciis gamoyeneba }

*insert* 3 (*insert* 2 (*insert* 1 [4]))

= { *insert* funqciis gamoyeneba }

*insert* 3 (*insert* 2 [1*,* 4])

= { *insert* funqciis gamoyeneba }

*insert* 3 [1*,* 2*,* 4]

= { *insert* funqciis gamoyeneba }

[1*,* 2*,* 3*,* 4]

**mravalargumentiani funqciebi**

amasTan erTad, mravalargumentiani funqciebi aseve SeiZleba ganisaz­R­vros rekursiis gamoyenebiT or an met argumentze. magaliTad, sabib­li­o­Te­ko *zip* funqcia, romelic iRebs Sesasvlelze or sias da wyvilebis Se­q­mnil sias gvibrunebs, Semdegi saxiT ganisazRvreba:

*zip* :: [*a*] *→* [*b*] *→* [(*a, b*)]

*zip* [] \_ = []

*zip \_* [] = []

*zip* (*x* : *xs*) (*y* : *ys*) = (*x , y*) : *zip xs ys*

magaliTad:

*zip* ['a'*,* 'b'*,* 'c'] [1*,* 2*,* 3*,* 4]

= { *zip* funqciis gamoyeneba }

('a'*,* 1) : *zip* ['b'*,* 'c'] [2*,* 3*,* 4]

= { *zip* funqciis gamoyeneba }

('a'*,* 1) : ('b'*,* 2) : *zip* ['c'] [3*,* 4]

= { *zip* funqciis gamoyeneba }

('a'*,* 1) : ('b'*,* 2) : ('c'*,* 3) : *zip* [] [4]

= { *zip* funqciis gamoyeneba }

('a'*,* 1) : ('b'*,* 2) : ('c'*,* 3) : []

= {Cawera siis saxiT }

[('a'*,* 1)*,* ('b'*,* 2)*,* ('c'*,* 3)]

yuradReba miaqcieT, rom *zip* funqciis gansazRvrebaSi saWiroa ori saba­zo gamosaxuleba, vinaidan argumentTa ori siidan nebismieri SeiZleba aRmoCndes carieli.

ganvixiloT ramdenime argumentze rekursiis kidev erTi magaliTi. saxel­dobr, sabiblioTeko *drop* funqcia, romelic siaSi anadgurebs elemen­te­bis mocemul raodenobas am siis dasawyisidan, Semdegi saxiT SeiZleba ga­ni­sa­zRvros:

*drop* :: *Int →* [*a*] *→* [*a*]

*drop* 0 *xs* = *xs*

*drop* (*n* + 1) [] = []

*drop* (*n* + 1) ( \_ : *xs*) = *drop n xs*

aq kvlav ori sabazo (sayrdeni, sayrdnobi) gamosaxulebaa saWiro: erTi \_ nulovani raodeno­bis elementTa gasanadgureblad siaSi, xolo meore \_ cariel siaSi erTi an ramdenime elementis ganadgurebis mcdelobisas.

**mravaljeradi rekursia**

funqciebi aseve SeiZleba ganisazRvros *mravaljeradi rekursiiT*, romel­Sic funqcia Tavis sakuTar gansazRvrebas araerTxel iyenebs. magaliTad, gavi­xsenoT fibonaCis ricxvTa 0*,* 1*,* 1*,* 2*,* 3*,* 5*,* 8*,* 13*, . . .* mimdevroba, ro­mel­Sic pirveli da meore ricxvia 0 da 1 Sesabamisad, xolo yoveli momdev­no ricxvi wina oris jams warmoadgens. haskelSi funqcia, romelic anga­ri­Sobs fibonaCis me-*n* ricxvs nebismieri mTeli *n* ≥ 0 ricxvisaTvis, Sem­deg­nairad SeiZleba ganisazRvros orjeradi rekursiis gamoyenebiT:

*fibonacci* :: *Int → Int*

*fibonacci* 0 = 0

*fibonacci*1 = 1

*fibonacci* (*n* + 2) = *fibonacci n* + *fibonacci* (*n* + 1)

kidev erT msgavs magaliTs mivmarToT. saxeldobr, gavixsenoT, rom pir­vel TavSi naCvenebi iyo siis swarafi daxarisxebis (mowesrigebis) quicksort saxelwodebiT cnobi­li meTodis ganxorcieleba. Sesabamisi funqciis gansazRvra Semdegi sa­xiT SeiZleba:

*qsort* :: *Ord a* ⇒[*a*] *→* [*a*]

*qsort* [] = []

*qsort* (*x* : *xs*) = *qsort smaller* ++ [*x*] ++ *qsort larger*

**where**

*smaller* = [*a | a ← xs, a* ≤ *x*]

*larger* = [*b | b ← xs, b > x*]

maSasadame, carieli sia ukve daxarisxebulia, xolo nebismieri aracarieli sia SeiZleba daxarisxdes, Tu mis Tavs movaTavsebT or sias Soris. isini warmoadgens mocemuli aracarieli siis kudis im elementebis daxarisxebis Sedegs, romlebic an naklebia am Tavze, an masze metia Sesabamisad.

**urTierTrekursia**

funqciebis gansazRvra aseve SeiZleba urTierTrekursiiT (ingl. *mutual re­cur­sion*), roca ori an meti funqcia mTlianad gansazRvrulia erT­ma­ne­Tis saSualebiT. magaliTad, ganvixiloT *even* (luwi ricxvi) da *odd* (ken­ti ricxvi) sabiblioTeko funqciebi. efeqturobis gazrdis mizniT es funq­ciebi, Cveulebriv, ganisazRvreba orze gayofis Sedegad miRebuli naS­Tis gamoyenebiT. magram arauaryofiTi mTeli ricxvebisaTvis maTi gan­sa­zRvra urTierTrekursiis saSualebiTac SeiZleba:

*even* :: *Int → Bool*

*even* 0 = *True*

*even* (*n* + 1) = *odd n*

*odd* :: *Int → Bool*

*odd* 0 = *False*

*odd* (*n* + 1) = *even n*

maSasadame, nuli \_ mTeli ricxvia, xolo nebismieri mkacrad dadebiTi ricxvi luwia, Tu misi winamavali ricxvi kentia, da nebismieri mkacrad dadebiTi ricxvi kentia, Tu misi winamavali ricxvi luwia. magaliTad:

*even* 4

= { *even* funqciis gamoyeneba }

*odd* 3

= { *odd* funqciis gamoyeneba }

*even* 2

= { *even* funqciis gamoyeneba }

*odd* 1

= { *odd* funqciis gamoyeneba }

*even* 0

= { *even* funqciis gamoyeneba }

*True*

amis msgavsad, funqciebi, romlebic irCevs elementebs siidan yvela luw da kent poziciaSi (am poziciis aTvlisas nulidan), SeiZleba Semdegi saxiT ganisazRvros Sesaamisad:

*evens* :: [*a*] *→* [*a*]

*evens* [] = []

*evens* (*x* : *xs*) = *x* : *odds xs*

*odds* :: [*a*] *→* [*a*]

*odds* [] = []

*odds* ( : *xs*) = *evens xs*

magaliTad:

*evens* "abcde"

= { *evens* funqciis gamoyeneba }

'a' : *odds* "bcde"

= { *odds* funqciis gamoyeneba }

'a' : *evens* "cde"

= { *evens* funqciis gamoyeneba }

'a' : ’c’ : *odds* "de"

= { *odds* funqciis gamoyeneba }

'a' : 'c' : *evens* "e"

= { *evens* funqciis gamoyeneba }

'a' : 'c' : 'e' : *odds* []

= { *odds* funqciis gamoyeneba }

'a' : 'c' : 'e' : []

= {Cawera siis saxiT }

"ace"

gavixsenoT, rom striqonebi haskelSi realurad agebulia rogorc simboloTa siebi. amrigad, "abcde" Canaweri mxolod abreviaturaa ['a'*,* 'b'*,* 'c'*,* 'd'*,* 'e'] gamosaxulebisaTvis da meti araferi.

**6.6 rekomendaciebi rekursiis sakiTxebze**

rekursiuli funqciebis gansazRvra velosipedis tarebas gvagonebs: gamo­i­yureba martivad, roca amas vinme sxva akeTebs, SeiZleba ganuxorcielebeli mo­ge­CvenoT, Tu amis gakeTebas damoukideblad Tavad moindomebT pirvelad, ma­gram martivi da bunebrivi xdeba praqtikis miRebis Semdeg. am nawilSi Cven rekomendacias viZleviT saerTod funqciaTa gansazRvris sakiTxebze da kerZod, rekursiul funqciaTa aRweris problemebze. amisaTvis miv­mar­TavT *xuTetapian process*, romelsac avsaxavT *sami konkretuli maga­li­Tis* safuZvelze.

**magaliTi I -** *product*

pirveli martivi magaliTis saxiT aRvweroT, rogor xdeba etapobrivad am TavSi adre mocemuli *product* sabiblioTeko funqciis gansazRvra, romelic angariSobs ricxvTa siis namravls.

**etapi 1: tipis aRwera**

dafiqreba tipebis Taobaze SeiZleba Zalian sasargeblo aRmoCndes funqciaTa gan­sa­­zRvrisas. ase rom funqciis tipis gansazRvra sakuTriv funqciis gansazRvris da­wyebamde \_ kargi da misasalmebeli Cvevaa. gansaxilveli magaliTis SemTxvevaSi vi­wyebT funqciis formiT:

*product* :: [*Int*] *→ Int*

am struqturidan Cans, rom *product* funqcia iRebs Sesasvlelze mTeli ricxvebis si­as da gamoaqvs gamosasvlelze ganmxoloebuli mTelricxva mniSvneloba. am ma­ga­­liTis msgavsad, sadac saubaria *Int* tipze, xSirad sasargebloa ganxilvis da­wye­ba swored aseTi martivi tipiT, romelic mogvianebiT SeiZleba dazustdes an ga­nzogaddes aucileblobis SemTxvevaSi.

**etapi 2: SemTxvevaTa CamoTvla**

argumentTa tipebis umravlesobisaTvis arsebobs rigi standartuli SemTxveva da ma­Ti ganxilva aucilebelia. saxeldobr, siebisaTvis standartuli SemTxvevebia ca­ri­eli sia da aracarieli siebi. amitom SegviZlia CavweroT Semdegi sqematuri gan­sazRvreba SablonTan Sedarebis gamoyenebiT:

*product* [] =

*product* (*n* : *ns*) =

arauaryofiTi mTeli ricxvebisaTvis standartuli SemTxvevebia 0 da *n*+ 1, lo­gi­kuri mniSvnelobebisaTvis aseTia *False* da *True* da ase Semdeg. mo­gvianebiT, ti­pe­bis msgavsad, SeiZleba dagvWirdes SemTxvevaTa da­zus­te­bac, magram maTi gan­xil­va sasargebloa standartuli SemTxvevebiT dai­wyos.

**etapi 3: martiv SemTxvevaTa gansazRvra**

mTeli ricxvebis namravli, roca am ricxvebis raodenoba nuls Seadgens, iZleva erTs, vinaidan erTiani ar arRvevs gamravlebis operaciaSi gamosaxulebis uc­vle­lo­bas. amitom carieli siis SemTxvevaSi bunebrivia ganisazRvros, rom:

*product* [] = 1

*product* (*n* : *ns*) =

am magaliTis msgavsad, martivi SemTxveva xSirad iZens ZiriTadi, sabazo SemTxvevis mniSvnelobas.

**etapi 4: sxva SemTxvevaTa gansazRvra**

rogor SeiZleba gamoviangariSoT mTeli ricxvebis aracarieli siis namravli? am etapze sasargeblo iqneba jer ganvixiloT Semadgeneli nawilebi, romlebic SeiZleba iqnes gamoyenebuli. aseTia, magaliTad, sakuTriv funqcia (*product*), argumentebi (*n* da *ns*) da Sesabamisi tipis (+, *−*, ∗ da a.S.) sabiblioTeko funqciebi. am SemTxvevaSi Cven mxolod vamravlebT pirvel mTel ricxvs da mTeli ricxvebis darCenili siis namravls:

*product* [] = 1

*product* (*n* : *ns*) = *n* ∗ *product ns*

am magaliTis msgavsad, sxva SemTxvevebi xSirad iZens rekursiul xasiaTs.

**etapi 5: ganzogadeba da gamartiveba**

mas Semdeg, rac funqcia ganisazRvreba zemoT aRwerili procesis gamoyenebiT, xSirad naTeli xdeba, rom igi SeiZleba ganzogaddes an gamartivdes. magaliTad, *product* funqcia damokidebuli ar aris im ricxvTa zust saxeze, romelTa mimarT igi gamoiyeneba. amitom misi tipi SeiZleba ganzogaddes da mTeli ricxvebidan savsebiT dasaSvebia nebismier ricxviT tipze gadasvla:

*product* :: *Num a* ⇒[a] *→ a*

rac Seexeba gamartivebas, me-7 TavSi vnaxavT, rom *product* funqciaSi gamoyenebuli rekursiis Sabloni inkafsulirebulia (e.i. kidev raRacas Seicavs sakuTar TavSi) *foldr* dasaxelebis sabiblioTeko funqciiT, romelsac *product* funqcia iyenebs. amitom *product* funqcia SeiZleba xelaxla ganisazRvros erTi gantolebiT

*product* = *foldr* (∗) 1

amrigad, saboloo gansazRvrebas *product* funqciisaTvis Semdegi saxe aqvs:

*product* :: *Num a* ⇒ [*a*] *→ a*

*product* = *foldr* (∗) 1

es aris *product* funqciis zusti ganmarteba A danarTSi mocemuli standartuli prelude failidan, imis gamoklebiT, rom efeqturobis mosazrebiT *foldr* funqciis gamoyeneba iq Canacvlebulia monaTesave *foldl* sabiblioTeko funqciiT, romelic aseve me-7 TavSia ganxiluli.

**magaliTi II - *drop***

axla ufro arsebiT magaliTze vuCvenoT, rogor SeiZleba xuTetapiani procesis gamoyenebiT avagoT sabiblioTeko *drop* funqciisaTvis adre mocemuli gansazRvreba. rogorc viciT, es funqcia anadgurebs siaSi elementebis mocemul raodenobas (am siis dasawyisidan).

**etapi 1: tipis aRwera**

daviwyoT tipiT, romelic gveubneba, rom *drop* funqcia iRebs Sesasvlelze mTel ricxvsa da garkveuli *a* tipis mniSvnelobaTa sias, xolo gamosasvlelze amave tipis sidideTa axali siis formirebas axorcielebs:

*drop* :: *Int →* [*a*] *→* [*a*]

yuradReba miaqcieT, rom am tipis gansazRvrebas oTxi Tavisebureba axa­si­a­Tebs, saxeldobr: 1. pirvel argumentad gamoyenebulia mTeli ricxvi da ara ufro zogadi ricxviTi tipi, rac simartivis uzrunvelsayofad keT­de­ba; 2. nacvlad imisa, rom funqcia Sesasvlelze iRebdes Tavis or ar­gu­ments wyvilis saxiT, meti moqnilobisaTvis Seqmnilia karingis ga­mo­ye­ne­bis SesaZlebloba (ix. punqti 3.6); 3. mTelricxva argumenti gan­Tav­se­bu­lia meore argumentamde, romelsac elementTa siis saxe aqvs, rac wa­kiT­xvis moxerxebulobas emsaxureba (*drop n xs* gamosaxuleba SeiZleba ase ikiT­xebodes, magaliTad: «*n* elementis amogdeba *xs* siidan»); 4. iqmneba funq­cia, romelic polimorfulia elementTa siis tipiT (gamoyenebis uni­ver­salobis uzrunvelsayofad).

**etapi 2: SemTxvevaTa CamoTvla**

vinaidan mTelricxva argumentisaTvis ori (0 da *n* + 1) standartuli Sem­Txve­va ar­sebobs, xolo argumentebis siisaTvis aseve or ([] da *x* : *xs*) gan­sa­kuTrebul mniS­vnelobasTan gvaqvs saqme, qvemoT mocemuli sqematuri Ca­na­weri gan­sa­zRvre­bi­sa­Tvis saerTo jamSi oTx gamosaxulebas moiTxovs:

*drop* 0 [] =

*drop* 0 (*x* : *xs*) =

*drop* (*n* + 1) [] =

*drop* (*n* + 1) (*x* : *xs*) =

**etapi 3: martiv SemTxvevaTa gansazRvra**

logikurad, nebismieri siis dasawyisidan misi nuli elementis amogdeba imave si­as iZleva. ase rom pirveli ori SemTxvevis gansazRvra uSualod xdeba:

*drop* 0 [] = []

*drop* 0 (*x* : *xs*) = *x* : *xs*

*drop* (*n* + 1) [] =

*drop* (*n* + 1) (*x* : *xs*) =

carieli siidan erTi an ramdenime elementis amogdebis mcdeloba dauSvebelia. amitom mesame SemTxveva unda iqnes gamotovebuli. magram es gamoiwvevs Secdomis gaCenas, Tu aseTi situacia warmoiqmneba. magram praqtikaSi miiReba gadawyvetileba Secdomis gaCenis asacileblad, risTvisac am SemTxvevaSi carieli siis dabruneba xdeba:

*drop* 0 [] = []

*drop* 0 (*x* : *xs*) = *x* : *xs*

*drop* (*n* + 1) [] = []

*drop* (*n* + 1) (*x* : *xs*) =

**etapi 4: sxva SemTxvevaTa gansazRvra**

rogor amovagdoT erTi an ramdenime elementi aracarieli siidan? erTiT naklebi raodenobis elementis martivi amoRebiT siis kudidan:

*drop* 0 [] = []

*drop* 0 (*x* : *xs*) = *x* : *xs*

*drop* (*n* + 1) [] = []

*drop* (*n* + 1) (*x* : *xs*) = *drop n xs*

**etapi 5: ganzogadeba da gamartiveba**

*drop* funqcia damokidebuli ar aris im mTeli ricxvis zust saxeze, ro­mel­sac igi pirvel argumentad iRebs. amitom xsenebuli ricxvis tipi Se­i­Zleba ganzogaddes da nebismier *Integral* mTelricxva tipad ga­mo­cxad­des, romlis standartul egzemplarebs[[2]](#footnote-2) *Int* da *Integer* tipebi war­mo­a­d­gens:

*drop* :: *Integral b* ⇒ *b →* [*a*] *→* [*a*]

magram efeqturobis mosazrebebiT aseTi ganzogadeba faqtobrivad ar xdeba stan­dar­tul prelude failSi, rogorc es aRiniSneboda punqtSi 3.9. gamartivebis Tval­sazrisiT ki pirveli ori gantoleba *drop* funqciisaTvis SeiZleba erT gan­to­lebad Caiweros, romelic gveubneba, rom nebismieri siidan nulovani rao­de­no­bis ele­ment­Ta amogdeba imave sias iZleva:

*drop* 0 *xs* = *xs*

*drop* (*n* + 1) [] = []

*drop* (*n* + 1) (*x* : *xs*) = *drop n xs*

garda amisa, *x* cvladi ukanasknel gantolebaSi SeiZleba SevcvaloT Cas­mis SabloniT, vinaidan es cvladi gantolebis tanSi ar gamoiyeneba:

*drop* 0 *xs* = *xs*

*drop* (*n* + 1) [] = []

*drop* (*n* + 1) ( \_ : *xs*) = *drop n xs*

amis msgavsad, Cndeba mosazreba, rom meore gantolebaSi *n* sidide unda iyos Canacvlebuli ( \_ ) CanaweriT, magram es dauSvebel da mcdar saxes aZlevs gansazRvrebas, vinaidan *n* + *k* formis Sablonebi moiTxovs, rom *n* si­di­de warmoadgendes cvlads. am SezRudvis Tavidan acileba Se­sa­Zle­be­li iqneba, Tu meore gantolebaSi mTlianad CavanacvlebT *n* + 1 Sablons ( \_ ) CanaweriT. magram es funqcis qcevasac Secvlis. magaliTad, *drop* (*−*1) [] gamosaxulebis gamoTvla am dros carieli siis SemTxvevaSic moxdeba, ma­Sin roca axla es Secdomas iwvevs, vinaidan ( \_ ) CanawerTan Se­sa­ba­mi­so­ba­Si nebismieri mTeli ricxvis moyvana SeiZleba, xolo (*n* + 1) Canawers mxo­lod iseTi mTeli ricxvebi Seesabameba, romlebic erTze naklebi ar aris (≥ 1).

dasasrul, saboloo gansazRvreba *drop* funqciisaTvis swored im saxes iRebs, romelic mas A danarTis prelude standartul failSi aqvs:

*drop* :: *Int →* [*a*] *→* [*a*]

*drop* 0 *xs* = *xs*

*drop* (*n* + 1) [] = []

*drop* (*n* + 1) ( \_ : *xs*) = *drop n xs*

**magaliTi III -** *init*

daskvniTi magaliTis saxiT vnaxoT gansazRvrebis ageba xuTetapian procesSi *init* sa­biblioTeko funqciisaTvis, romelic aracarieli siis ukanasknel elements ana­dgurebs.

**etapi 1: tipis aRwera**

daviwyoT tipis aRweriT, romelic gveubneba, rom *init* funqcia iRebs Se­sa­s­vlel­ze garkveuli tipis mniSvnelobaTa sias da agebs aseTive mni­S­vne­lo­bebis sxva si­as:

*init* :: [*a*] *→* [*a*]

**etapi 2: SemTxvevaTa CamoTvla**

vinaidan carieli sia ar aris dasaSvebi argumenti *init* funqciisaTvis, gan­sa­z­Rvre­bis qvemoT naCvenebi sqematuri Canaweri SablonTan Sedarebis gamo­yenebiT mxo­lod erTi SemTxvevis miTiTebas moiTxovs:

*init* (*x* : *xs*) =

**etapi 3: martiv SemTxvevaTa gansazRvra**

maSin, roca or wina magaliTSi es etapi savsebiT cxadi iyo, *init* funqciis Sem­Txve­vaSi odnav meti dafiqreba dagvWirdeba. logikurad, ukanaskneli elementis amo­Reba erTelementiani siidan cariel sias iZleva. amitom SegviZlia mcvelis Se­motana am martivi SemTxvevisaTvis:

*init* (*x* : *xs*) *| null xs* = []

*| otherwise* =

gavixsenoT, rom sabiblioTeko *null* funqcia adgens carielia sia, Tu ara.

**etapi 4: sxva SemTxvevaTa gansazRvra**

rogor SeiZleba ukanaskneli elementis amoReba siidan, romelSic sul cota ori elementi mainc aris? Tavis martivi SenarCunebiT da ukanaskneli elementis amoRebiT kudidan:

*init* (*x* : *xs*) *| null xs* = []

*| otherwise* = *x* : *init xs*

**etapi 5: ganzogadeba da gamartiveba**

*init* funqciisaTvis tipi ukve imdenad zogadia, ramdenadac es Se­sa­Zle­be­lia, magram miuxedavad amisa, Tavad gansazRvreba SeiZleba gamartivdes Sa­blonTan SedarebiT mcvelebis nacvlad da pirvel gantolebaSi Casmis Sa­blonis gamoyenebiT da ara cvladis:

*init* :: [*a*] *→* [*a*]

*init* [ \_ ] = []

*init* (*x* : *xs*) = *x* : *init xs*

am SemTxvevaSic swored aseTia *init* funqciis gansazRvreba prelude standartul failSi.

1. xeliT daxarisxebis martivi da Zalian araefeqturi meTodi, roca monacemTa morigi elementi Tavsdeba siis saWiro adgilze siis arsebul elementebTan Sedarebis Semdeg. [↑](#footnote-ref-1)
2. klasis egzemplari (ingl. instance) \_ konkretuli obieqti mocemuli klasis obieqtebis si­mravlidan. sistemaSi, Cveulebriv, sxvadasxva klasis mravali egzemplari funqcio­ni­rebs. erTi klasis yvela egzemplars operaciaTa erTnairi nakrebi aqvs. [↑](#footnote-ref-2)